

GCE

Mathematics

Advanced GCE

Unit 4730: Mechanics 3

Mark Scheme for June 2011

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1	[$5\cos\theta - 4 = 0$] $\cos\theta = 0.8$ [$I = 0.3(5\sin\theta - 0)$ or $\sin\theta = I \div (0.3 \times 5)$] I = 0.9	M1 A1 M1 A1 [4]	For using $v_x - u_x = 0$ or for a triangle sketched with sides $I/0.3$, 4 and 5 with angles θ and 90° opposite I/m and 5 respectively. AG For using $I = m(\Delta v)$ in 'y' direction or $I = \sqrt{\left(\left(0.3 \times 5\right)^2 - \left(0.3 \times 4\right)^2\right)}$ M1
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2 i	(1.0. 2.0) P. (2.2. 0.0) 200. 1.5.400	M1	For taking moments about <i>C</i> for the whole for M1 need 3 terms; allow 1 sign error and/or 1 length error and/or still including sin/cos
	$(1.8 + 3.2)R_B = (3.2 + 0.9)x300 + 1.6x400$ Force exerted on <i>AB</i> is 374 N	A1 A1	
	Force exerted on AC is 326 N	B1 [4]	or for taking moments about <i>B</i> for whole $(1.8 + 3.2)R_C = (1.8 + 1.6)x400 + 0.9x300$ giving force on <i>AC</i> first: M1A1A1A1
ii		M1	For taking moments about A for AB for M1 need 3 terms, allow 1 sign error and/or 1 length error and/or still including sin/cos
	0.9x300 + 1.2T = 1.8x374	A1	or moments about A for AC
	Tension is 336 N	A1 [3]	1.6x400 + 1.2T = 3.2x326
iii	Horizontal component is 336 N to the left $[Y = 374 - 300]$ Vertical component is 74 N downwards	B1ft M1 A1ft [3]	For resolving forces on AB vertically

Give credit for part (ii) done on the way to part (i) if not contradicted in (ii).

3 i	$0.25(dv/dt) = -0.2v^{2}$ $0.25 \int v^{-2} dv = -0.2t(+C)$	M1 dep M1	For using Newton's second law with $a = dv/dt$. Allow sign error and/or omitting mass For separating variables and attempting to integrate (ie get v^{-1} and t).
	$-v^{-1}/4 = -t/5 + C$ $[1/4v = t/5 + 1/20]$ $v = \frac{5}{4t+1} \text{ oe}$	A1 M1 A1 [5]	For using $v(0) = 5$ to obtain C
ii	$x = (5/4)\ln(4t + 1) \ (+B)$ Subst $v = 0.2$ in (i) to find t Obtain $x(6)$ (= 1.25 ln25 oe (4.02359)) Average speed is 0.671 ms ⁻¹	M1 A1 M1 M1 A1 [5]	For using $v = dx/dt$ and integrating Implied by $t = 6$ May be written as $\frac{5}{12} \ln 5$
	Alternatively $\ln v = -0.8x + B$ Subst $v = 0.2$ in (i) to find t Obtain $x(0.2)$ (= 1.25 ln(5/0.2) oe (4.0239)) Average speed is 0.671 ms ⁻¹	M1 A1 M1 M1 A1	For using $mv(dv/dx) = -0.2v^2$, separating variables and integrating. Allow sign error and/or omitting mass. Implied by $t = 6$ May be written as $\frac{5}{12} \ln 5$

4 i	[-0.2x2 $\ddot{\theta} = 0.2g\sin\theta$] $\frac{d^2\theta}{dt^2} = -4.9\sin\theta$ For small θ , $\sin\theta \approx \theta$ and $\ddot{\theta} = -4.9\theta$ represents SHM	M1 A1 B1 [3]	For using Newton's second law transversely. Allow sign error and/or sin/cos error and/or missing 0.2, g or l. AG
ii	θ = 0.15cos($\sqrt{4.9} t$) oe t = 1.04 at first occasion t = 1.80 at second occasion	M1 A1 A1 M1 A1 [5]	For using $\theta = A\cos(nt)$ or $A\sin(nt + \varepsilon)$. Allow \sin/\cos confusion for using $t_1 + t_2 = 2\pi/n$
iii	Angular speed is (-) 0.297 rads s ⁻¹ Linear speed is (-) 0.594ms ⁻¹	M1 A1 A1ft [3]	For using $\dot{\theta} = -An \sin(nt)$ oe. Allow sign error and/or ft from θ in (ii).

In (ii) & (iii) allow M marks if angular displacement/speed has been confused with linear.

5	$[\sin \gamma = 0.96 \div 1.2]$	M1	For using $v_B \sin \gamma = u_B \sin \beta$
i	$\sin \gamma = 0.8$	A1	
	·	[2]	
ii		M1	For using the principle of conservation of momentum. Allow sign error and/or
		****	$u_A \cos \alpha$ (instead of 2) for M1.
	$(m)2 - (m)u_B\cos\beta = (m)v_B\cos\gamma$	A1	allow $u_A \cos \alpha$ (instead of 2) for A1
		M1	For eliminating u_B or v_B . Allow with cos
	$2 = v_B(0.6 + 0.28 \div 1.2)$	A1	Or $2 = 0.28u_B + 0.72u_B$
	$v_B = 2.4, u_B = 2$	A1 [5]	
iii	$[(2 + u_B \cos \beta) e = v_B \cos \gamma]$	M1	For applying Newton's exp'tal law. Allow sign error and/or $u_A\cos\alpha$ (instead of
		A 1.64	2) for M1.
	$(2 + 2 \times 0.28)e = 2.4 \times 0.6$	A1ft	ft u_B and v_B only
	$e = \frac{9}{6}$ or 0.5625	A1	
	$(2 + 2 \times 0.28)e = 2.4 \times 0.6$ $e = \frac{9}{16} \text{ or } 0.5625$	[3]	
iv	_		For using $\frac{1}{2}(m)v^2 = 6.5(m)$ and
	$[(y\text{-component})^2 = 13 - 4]$	M1	$(y$ -component) ² = v^2 - 2^2 . Allow 1 slip.
	$v_A = (y\text{-component})_{\text{before}} = 3$	A1	
		[2]	

6		M1	For using PE gain = $W(h_Y - h_X)$
i	PE gain = $6 \times 0.8 (\sqrt{3}/2 - 1/\sqrt{2})$		
			Shown fully, with no slips
	$=2.4(\sqrt{3}-\sqrt{2})$	A1	AG
		M1	For using EE loss = $\lambda (e_x^2 - e_y^2)/2l$. Allow
	EE loss = $\frac{9}{2(\pi/10)}$ [(0.8 $\pi/4$ - $\pi/10$) ² –		slips for M1.
	$(0.8 \pi/6 - \pi/10)^2$	A1	Fully correct
	EE loss = $45 \pi [(0.2 - 0.1)^2 - (0.4 - 0.3)^2 \div 9]$	A1	No slips in simplification
	$= 5 \pi (9x0.01 - 0.01) = 40 \pi / 100 = 0.4 \pi J$	[5]	AG
ii			
	$T = 9 (0.8 \pi / 6 - \pi / 10) \div (\pi / 10)$	B1	
		M1	For attempting to show that
	$W\sin\theta - T = 6 \times \sin(\pi/6) - 90 \times (0.2 \div 6) = 0$		Wsin θ - $T = 0$ at Y by subst $\theta = \pi/6$
	→ transverse acceleration is zero	A1	AG No slips
		M1	For using KE gain = EE loss – PE gain at
			Y. Need 3 terms, allow sign errors and/or
	$\frac{1}{2}(6/9.8)v^2 = 0.4 \pi - 2.4(\sqrt{3} - \sqrt{2})$	A1	g omitted.
	Maximum speed is 1.27 ms ⁻¹	A1	
		[6]	

7 i		M1	For using the principle of conservation of energy. Allow sign error, sin/cos; need 3 terms.
	$\frac{1}{2}mv^2 = \frac{1}{2}m5.6^2 - mg0.8(1 - \cos\theta)$	A1	
	$v^2 = 15.68(1 + \cos\theta)$	A1	AG No slips
	$T - mg\cos\theta = mv^2/r$	M1	For using Newton's second law. Allow sign error and/or sin/cos and/or <i>m</i> omitted
	$[T - 0.3g\cos\theta = 0.3x15.68(1 + \cos\theta)/0.8]$	A1 M1 A1	For substituting for v^2
	Tension is $2.94(3\cos\theta + 2)$ N oe	[7]	
ii	θ is 131.8° (or 2.3 rads) Accept 132° (exact)	M1 A1	For putting $T = 0$ and attempting to solve accept $\theta = \cos^{-1}(-2/3)$
	v is 2.29	B1 [3]	$\sqrt{15.68/3}$ exact
iii	[speed = $ v \cos(180 - \theta) = \sqrt{15.68/3} \times (2/3)$]	M1	For using 'speed at max. height = horiz. comp. of vel. when string becomes slack'
	Speed at greatest height is 1.52 ms^{-1} $0.3gH = \frac{1}{2} \ 0.3(5.6^2 - 1.52^2)$	A1	For using the principle of conservation of
	0.3gH = ½ 0.3(5.6 – 1.52) Greatest height is 1.48 m	M1 A1 [4]	energy 40/27 exact
	ALTERNATIVE for (iii)	<u>r.</u> :1	
	$[0 = 2.286^{2} \times (1-4/9) -19.6y,$ $H = 0.8(1 + 2/3) + y]$ $H = 1.3333 + 0.1481 (4/3 + 4/27)$ Greatest height is 1.48 m (40/27)	M1 A1	For using $0^2 = \dot{y}^2 - 2gy$ and $H = 0.8\{1 + \cos(180 - \theta)\} + y$
	[$\frac{1}{2}m(2.286^2 - \text{speed}^2) = mg \times 0.1481$ speed ² = 2.286 ² - 19.6 × 0.1481] or [$\frac{1}{2}m(5.6^2 - \text{speed}^2) = mg \times 1.481$ speed ² = 5.6 ² - 19.6 × 1.481] Speed at greatest height is 1.52 ms ⁻¹	M1 A1	For using the principle of conservation of energy

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