GCE

# Mathematics 

Advanced GCE
Unit 4730: Mechanics 3

## Mark Scheme for June 2011

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| 1 | $\begin{aligned} & {[5 \cos \theta-4=0]} \\ & \cos \theta=0.8 \\ & {[I=0.3(5 \sin \theta-0) \text { or } \sin \theta=I \div(0.3 \times 5)]} \\ & I=0.9 \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> [4] | For using $v_{x}-u_{x}=0$ <br> or for a triangle sketched with sides $I / 0.3,4$ and 5 with angles $\theta$ and $90^{\circ}$ opposite $I / m$ and 5 respectively. <br> AG <br> For using I $=m(\Delta v)$ in ' $y$ ' direction or $I=\sqrt{\left((0.3 \times 5)^{2}-(0.3 \times 4)^{2}\right)} \quad$ M1 |
| :---: | :---: | :---: | :---: |


| 2 | $(1.8+3.2) R_{B}=(3.2+0.9) \times 300+1.6 \times 400$ <br> Force exerted on $A B$ is 374 N <br> Force exerted on $A C$ is 326 N | M1 <br> A1 <br> A1 <br> B1 <br> [4] | For taking moments about $C$ for the whole for M1 need 3 terms; allow 1 sign error and/or 1 length error and/or still including sin/cos <br> or for taking moments about $B$ for whole $(1.8+3.2) R_{C}=(1.8+1.6) \times 400+0.9 \times 300$ giving force on $A C$ first: M1A1A1A1 |
| :---: | :---: | :---: | :---: |
| ii | $\begin{aligned} & 0.9 \times 300+1.2 T=1.8 \times 374 \\ & \text { Tension is } 336 \mathrm{~N} \end{aligned}$ | M1 <br> A1 <br> A1 <br> [3] | For taking moments about $A$ for $A B$ for M1 need 3 terms, allow 1 sign error and/or 1 length error and/or still including sin/cos or moments about $A$ for $A C$ $1.6 \times 400+1.2 T=3.2 \times 326$ |
| iii | Horizontal component is 336 N to the left $[Y=374-300]$ <br> Vertical component is 74 N downwards | B1ft <br> M1 <br> A1ft <br> [3] | For resolving forces on $A B$ vertically |

Give credit for part (ii) done on the way to part (i) if not contradicted in (ii).

| 3 | $\begin{aligned} & 0.25(\mathrm{~d} v / \mathrm{d} t)=-0.2 v^{2} \\ & 0.25 \int v^{-2} d v=-0.2 t(+C) \\ & -v^{-1} / 4=-t / 5+C \\ & {[1 / 4 v=t / 5+1 / 20]} \\ & v=\frac{5}{4 t+1} \text { oe } \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { dep } \\ \text { M1 } \\ \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ {[5]} \end{gathered}$ | For using Newton's second law with $a=\mathrm{d} v / \mathrm{d} t$. Allow sign error and/or omitting mass <br> For separating variables and attempting to integrate (ie get $v^{-1}$ and $t$ ). <br> For using $v(0)=5$ to obtain $C$ |
| :---: | :---: | :---: | :---: |
| ii | $x=(5 / 4) \ln (4 t+1)(+B)$ <br> Subst $v=0.2$ in (i) to find $t$ <br> Obtain $x(6)(=1.25 \ln 25$ oe (4.02359...)) <br> Average speed is $0.671 \mathrm{~ms}^{-1}$ | M1 A1 M1 M1 A1 [5] | For using $v=\mathrm{d} x / \mathrm{d} t$ and integrating Implied by $t=6$ <br> May be written as $\frac{5}{12} \ln 5$ |
|  | Alternatively $\ln v=-0.8 x+B$ <br> Subst $v=0.2$ in (i) to find $t$ <br> Obtain $x(0.2)(=1.25 \ln (5 / 0.2)$ oe (4.0239...)) <br> Average speed is $0.671 \mathrm{~ms}^{-1}$ | $\begin{aligned} & \text { M1 } \\ & \\ & \text { A1 } \\ & \text { M1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ | For using $m v(\mathrm{~d} v / \mathrm{d} x)=-0.2 v^{2}$, separating variables and integrating. Allow sign error and/or omitting mass. <br> Implied by $t=6$ <br> May be written as $\frac{5}{12} \ln 5$ |


| 4 | $\begin{aligned} & {[-0.2 \times 2 \ddot{\theta}=0.2 g \sin \theta]} \\ & \frac{d^{2} \theta}{d t^{2}}=-4.9 \sin \theta \end{aligned}$ <br> For small $\theta, \sin \theta \approx \theta$ and $\ddot{\theta}=-4.9 \theta$ represents SHM | M1 <br> A1 <br> B1 <br> [3] | For using Newton's second law transversely. Allow sign error and/or $\sin /$ cos error and/or missing $0.2, g$ or $l$. AG |
| :---: | :---: | :---: | :---: |
| ii | $\theta=0.15 \cos (\sqrt{4.9} t)$ oe $t=1.04$ at first occasion <br> $t=1.80$ at second occasion | M1 A1 A1 M1 A1 [5] | For using $\theta=A \cos (n t)$ or $A \sin (n t+\varepsilon)$. Allow sin/cos confusion <br> for using $t_{1}+t_{2}=2 \pi / n$ |
| iii | Angular speed is (-) $0.297 \mathrm{rads} \mathrm{s}^{-1}$ <br> Linear speed is (-) $0.594 \mathrm{~ms}^{-1}$ | M1 <br> A1 <br> A1ft <br> [3] | For using $\dot{\theta}=-A n \sin (n t)$ oe. Allow sign error and/or ft from $\theta$ in (ii). |

In (ii) \& (iii) allow M marks if angular displacement/speed has been confused with linear.

| $5$ | $\begin{aligned} & {[\sin \gamma=0.96 \div 1.2]} \\ & \sin \gamma=0.8 \end{aligned}$ | $\begin{array}{\|c\|} \hline \text { M1 } \\ \text { A1 } \\ {[2]} \end{array}$ | For using $v_{B} \sin \gamma=u_{B} \sin \beta$ |
| :---: | :---: | :---: | :---: |
| ii | $\begin{aligned} & (m) 2-(m) u_{B} \cos \beta=(m) v_{B} \cos \gamma \\ & 2=v_{B}(0.6+0.28 \div 1.2) \\ & v_{B}=2.4, u_{B}=2 \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> A1 <br> [5] | For using the principle of conservation of momentum. Allow sign error and/or $u_{A} \cos \alpha$ (instead of 2) for M1. allow $u_{A} \cos \alpha$ (instead of 2 ) for A1 <br> For eliminating $u_{B}$ or $v_{B}$. Allow with cos Or $2=0.28 u_{B}+0.72 u_{B}$ |
| iii | $\begin{aligned} & {\left[\left(2+u_{B} \cos \beta\right) e=v_{B} \cos \gamma\right]} \\ & (2+2 \times 0.28) e=2.4 \times 0.6 \\ & e=\frac{9}{16} \text { or } 0.5625 \end{aligned}$ | M1 <br> A1ft <br> A1 <br> [3] | For applying Newton's exp'tal law. <br> Allow sign error and/or $u_{A} \operatorname{Cos} \alpha$ (instead of <br> 2) for M1. <br> ft $u_{B}$ and $v_{B}$ only |
| iv | $\begin{aligned} & {\left[(y \text {-component })^{2}=13-4\right]} \\ & v_{A}=(y \text {-component })_{\text {before }}=3 \end{aligned}$ | M1 <br> [2] | For using $1 / 2(m) v^{2}=6.5(m)$ and $(y \text {-component })^{2}=v^{2}-2^{2}$. Allow 1 slip. |


| 6 | $\begin{aligned} & \text { PE gain }=6 \times 0.8(\sqrt{3} / 2-1 / \sqrt{2}) \\ & =2.4(\sqrt{3}-\sqrt{2}) \end{aligned} \quad \begin{array}{r} \text { EE loss }=\frac{9}{2(\pi / 10)}\left[(0.8 \pi / 4-\pi / 10)^{2}-\right. \\ \text { EE loss }=45 \pi\left[(0.2-0.1)^{2}-(0.8 \pi / 6-\pi / 10)^{2}\right] \\ =5 \pi(9 \times 0.01-0.01)=40 \pi / 100=0.4 \pi \mathrm{~J}) \end{array}$ | A1 <br> M1 <br> A1 <br> A1 <br> [5] | For using PE gain $=W\left(h_{Y}-h_{X}\right)$ <br> Shown fully, with no slips <br> AG <br> For using EE loss $=\lambda\left(e_{X}{ }^{2}-e_{Y}^{2}\right) / 2 l$. Allow slips for M1. <br> Fully correct <br> No slips in simplification AG |
| :---: | :---: | :---: | :---: |
| ii | $T=9(0.8 \pi / 6-\pi / 10) \div(\pi / 10)$ <br> $W \sin \theta-T=6 \times \sin (\pi / 6)-90 \times(0.2 \div 6)=0$ <br> transverse acceleration is zero $1 / 2(6 / 9.8) v^{2}=0.4 \pi-2.4(\sqrt{3}-\sqrt{2})$ <br> Maximum speed is $1.27 \mathrm{~ms}^{-1}$ | B1 <br> M1 <br> A1 <br> M1 <br> A1 <br> A1 <br> [6] | For attempting to show that $W \sin \theta-T=0$ at $Y$ by subst $\theta=\pi / 6$ AG No slips For using KE gain = EE loss - PE gain at Y. Need 3 terms, allow sign errors and/or g omitted. |


| 7 | $\begin{aligned} & 1 / 2 m v^{2}=1 / 2 m 5.6^{2}-m g 0.8(1-\cos \theta) \\ & v^{2}=15.68(1+\cos \theta) \\ & T-m g \cos \theta=m v^{2} / r \\ & {[T-0.3 g \cos \theta=0.3 \times 15.68(1+\cos \theta) / 0.8]} \end{aligned}$ $\text { Tension is } 2.94(3 \cos \theta+2) \mathrm{N} \text { oe }$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { A1 } \\ \text { M1 } \\ \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ {[7]} \end{gathered}$ | For using the principle of conservation of energy. Allow sign error, sin/cos; need 3 terms. <br> AG No slips <br> For using Newton's second law. Allow sign error and/or sin/cos and/or $m$ omitted <br> For substituting for $v^{2}$ |
| :---: | :---: | :---: | :---: |
| ii | $\theta$ is $131.8^{\circ}$ (or 2.3 rads) Accept $132^{\circ}$ (exact) $v$ is 2.29 | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { B1 } \\ {\left[\begin{array}{l} 3] \end{array} .\right.} \end{gathered}$ | For putting $T=0$ and attempting to solve accept $\theta=\cos ^{-1}(-2 / 3)$ <br> $\sqrt{15.68 / 3}$ exact |
| iii | $\begin{aligned} & {[\text { speed }=\|v \cos (180-\theta)\|}= \\ &\sqrt{15.68 / 3} \times(2 / 3)] \end{aligned}$ <br> Speed at greatest height is $1.52 \mathrm{~ms}^{-1}$ $0.3 g H=1 / 20.3\left(5.6^{2}-1.52 . . .^{2}\right)$ <br> Greatest height is 1.48 m | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ [4] | For using ‘speed at max. height = horiz. comp. of vel. when string becomes slack' <br> For using the principle of conservation of energy 40/27 exact |
|  | ALTERNATIVE for (iii) $\begin{array}{\|l} {\left[0=2.286 . .^{2} \times(1-4 / 9)-19.6 y,\right.} \\ H=0.8(1+2 / 3)+y] \\ H=1.3333 . .+0.1481 \ldots(4 / 3+4 / 27) \end{array}$ <br> Greatest height is 1.48 m (40/27) <br> [ $1 / 2 m\left(2.286 \ldots{ }^{2}-\right.$ speed $\left.^{2}\right)=m g \times 0.1481 \ldots$ <br> speed $^{2}=2.286$.. $^{2}-19.6 \times 0.1481 \ldots$... ] or <br> $\left[1 / 2 m\left(5.6^{2}-\right.\right.$ speed $\left.^{2}\right)=m g \times 1.481 \ldots$ <br> speed $\left.^{2}=5.6^{2}-19.6 \times 1.481 \ldots . \quad\right]$ <br> Speed at greatest height is $1.52 \mathrm{~ms}^{-1}$ | M1 <br> A1 <br> M1 <br> A1 | For using $0^{2}=\dot{y}^{2}-2 g y$ and $H=0.8\{1+\cos (180-\theta)\}+y$ <br> For using the principle of conservation of energy |

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